Risk Neutral Probability

$$P = \frac{e^{rt} - d}{u - d}$$

$e^{rt} = e^{0.036} = 1.037$

$u = \frac{592}{421} = 1.4062$

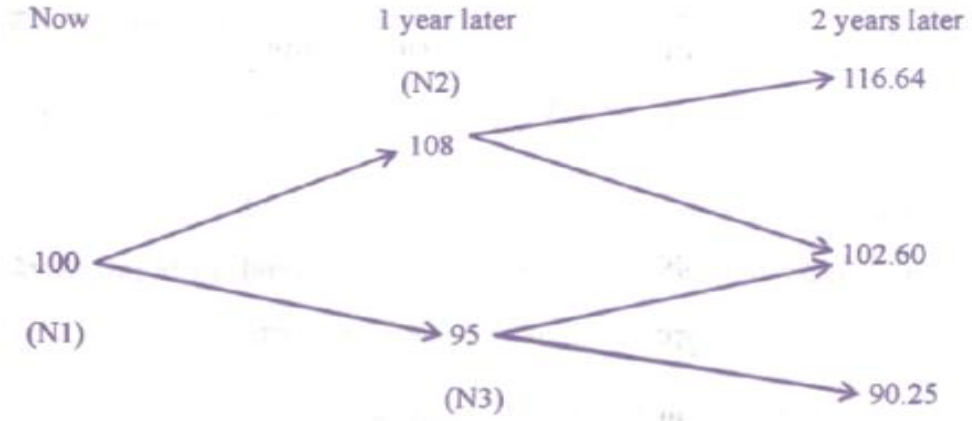
$d = \frac{411}{421} = 0.9762$

$$P = \frac{1.037 - 0.9762}{1.4062 - 0.9762} = 0.1414$$

TWO PERIOD BINOMIAL MODELS

Question – 10

A two year tree for a share of stock in ABC Ltd., is as follows:



Consider a two years American call option on the stock of ABC Ltd., with a strike price of ₹ 98. The current price of the stock is ₹ 100. Risk free return is 5 per cent per annum with a continuous compounding and $e^{0.05} = 1.05127$. Assume two time periods of one year each.

Using the Binomial Model, calculate:

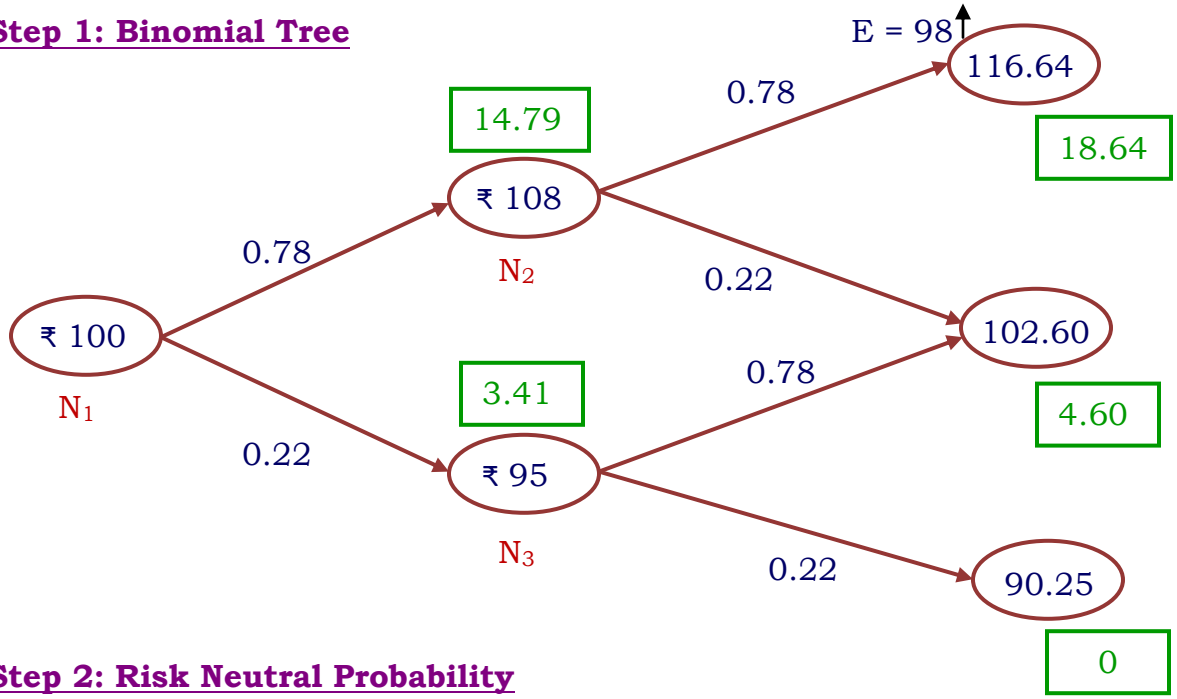
(i) The probability of price moving up and down;

(ii) Expected pay offs at each nodes i.e. N1, N2 and N3 (round off upto 2 decimal points).

(Exam Nov – 2020) (8 Marks)

Solution:

Step 1: Binomial Tree



Step 2: Risk Neutral Probability

$$P = \frac{e^{rt} - d}{u - d}$$

$$P = \frac{1.05127 - 0.95}{1.08 - 0.95}$$

$$= 0.78$$

Step 3: Value of Option

N_2

$$C_0 = \frac{(18.64 \times 0.78) + (4.60 \times 0.22)}{1.05127}$$

$$= 14.79$$

Intrinsic Value = 108 – 98

= ₹ 10

Hence, value of option at node 2 is ₹ 14.79

N_3

$$C_0 = \frac{(4.60 \times 0.78) + (0 \times 0.22)}{1.05127}$$

$$= 3.41$$

Intrinsic Value = 0

Hence, value of option at node 3 is ₹ 3.41

N_1

$$C_0 = \frac{(14.79 \times 0.78) + (3.41 \times 0.22)}{1.05127}$$

$$= 11.69$$

Intrinsic Value = 2

Hence, value of call today is ₹ 11.69

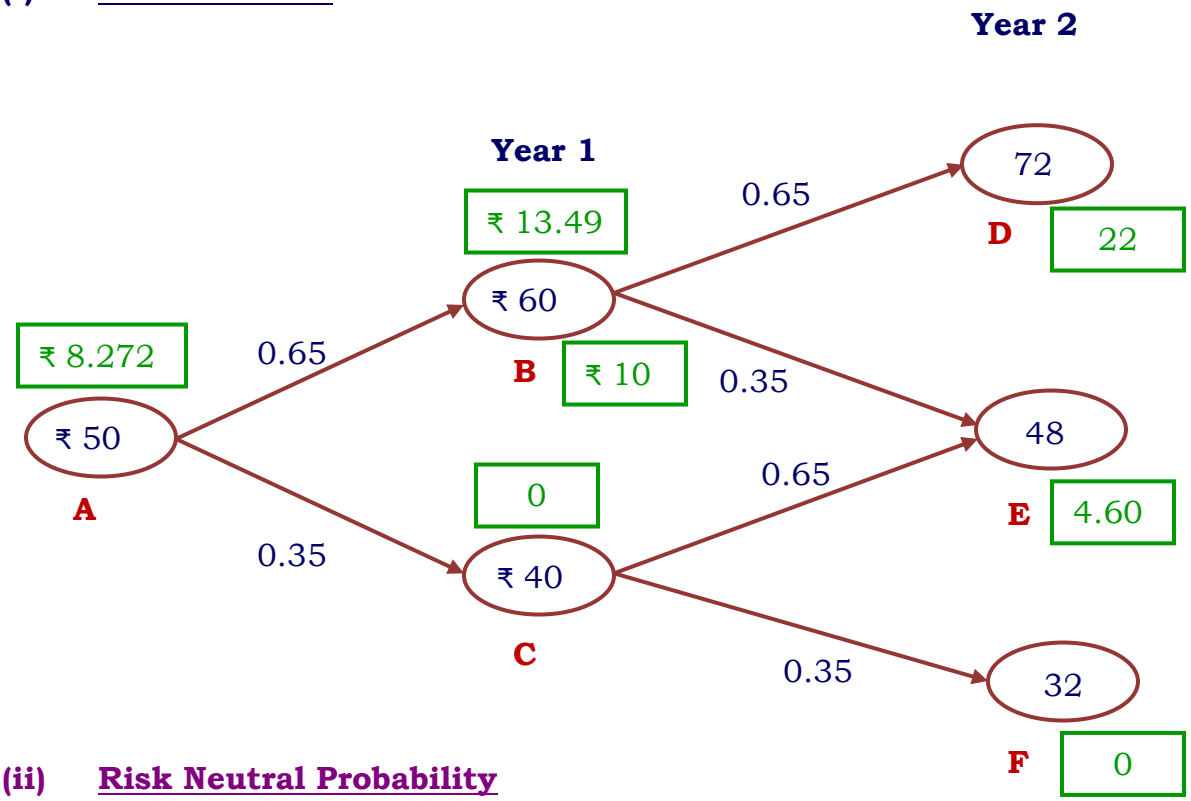
Question – 11

Consider a two-year call option with a strike price of ₹ 50 on a stock the current price of which is also ₹ 50. Assume that there are two-time periods of one year and in each year the stock price can move up or down by equal percentage of 20%. The risk-free interest rate is 6%. Using binominal option model, calculate the probability of price moving up and down. Also draw a two-step binomial tree showing prices and payoffs at each node.

(SM TYK – 23)

Solution:

(i) Binomial Tree



(ii) Risk Neutral Probability

R = 6%, u = 1.20, d = 0.80

$$P = \frac{R-d}{u-d} = \frac{1.06-0.80}{1.20-0.80} = 0.65$$

(iii) Value of Option

Node B $= \frac{(22 \times 0.65) + (0 \times 0.35)}{1.06} =$ ₹ 13.49

P.V. of Expected Payoff = ₹ 13.49

Intrinsic Value = ₹ 10

Value of option at Node B (Higher) = ₹ 13.49

Node C Value of Option = 0

Node A $= \frac{(₹\ 13.49 \times 0.65) + (0 \times 0.35)}{1.06} =$ ₹ 8.272

Intrinsic Value = 50 – 50 = 0

Hence Value of Call Option today is ₹ 8.272

DELTA HEDGING BINOMIAL MODEL

Question – 12

Mr. Dayal is interested in purchasing equity shares of ABC Ltd. which are currently selling at ₹ 600 each. He expects that price of share may go upto ₹ 780 or may go down to ₹ 480 in three months. The chances of occurring such variations are 60% and 40% respectively. A call option on the shares of ABC Ltd. can be exercised at the end of three months with a strike price of ₹ 630.

(i) What combination of share and option should Mr. Dayal select if he wants a perfect hedge?

(ii) What should be the value of option today (the risk free rate is 10% p.a.)?

(iii) What is the expected rate of return on the option?

(SM TYK – 22)

Solution:

Given: S = ₹ 600

us = ₹ 780

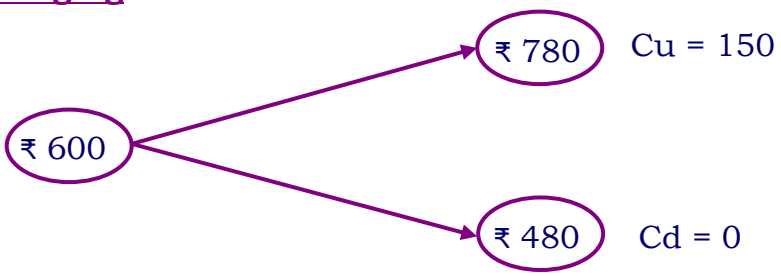
ds = ₹ 480

Period = 3 Months

E = ₹ 630

Option = Call

Delta Hedging



(i) Combination of Shares & Options

$$\text{Delta of Call} = \frac{Cu - Cd}{us - ds}$$

$$= \frac{150 - 0}{₹\,780 - ₹\,480} = 0.5$$

Write a Call Option & buy 0.5 share today for perfect hedge.

(ii) Cash Flow

(1) Price = ₹ 780

Call Option Exercised	= (150)
Sell Shares (₹ 780 × 0.5)	= 390
Cash Inflows	= 240

(2) Price = ₹ 480

Call Option Lapsed	= 0
Sell Shares (₹ 480 × 0.5)	= 240
Cash Inflows	= 240
Present Value Cash Inflow	$= \frac{240}{1.025}$
	= 234.15
Value of Call	= 0.5 × 600 – 234.15
	= ₹ 65.85

(3) Cost of option = 65.85

Expected Profit = (150 × 0.6) + (0 × 0.5)

= ₹ 90

$$\text{Expected Rate of Return} = \frac{90 - 65.85}{65.85} \times 100$$

= 36.67%

Question – 13

Following is the information available pertaining to shares of Omni Ltd.:

Current Market Price (₹)	₹ 420.00
Strike Price (₹)	₹ 450.00
Maximum Price (₹) expected in next 3 months' time	₹ 525.00
Minimum Price (₹) expected in next 3 months' time	₹ 378.00
Continuously Compounded Rate of Interest (p.a.) (%)	8.00%
e^{rt}	1.0202

From the above:

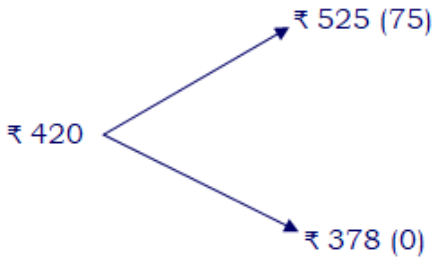
(i) Calculate the 3 months call option by using Binomial Method and Risk Neutral Method. Are the calculated values under both the models are same?

(ii) State also clearly the basis of Valuation of options under these models.

(Exam Nov – 2023) (8 Marks)

Solution:

(i) (1) Call Option value using Binomial Model



$$\Delta = \frac{₹\ 75 - 0}{₹\ 525 - ₹\ 378} = 0.51$$

Delta of Call = 0.51 means write 1 call & buy 0.51 share.

Initial Cash Outflows = 420 × 0.51= 214.20

Cash Flows on Maturity

If price 525

Sell share (525 × 0.51) = 267.75

Call Exercise = (75)

Cash Inflows = 192.75

If price 378

Sell Share (378 × 0.51) = 192.78

Call Lapse = 0

Cash Inflows = 192.78

P.V. off payoff $= \frac{192.78}{1.0202} = 188.96$

C_0 = 214.20 – 188.96 = ₹ 25.24

(2) Value of Call Option using Risk Neutral Method

Let "P' be the probability of Price increase, then

p × 525 + (1 – p) × 378 = 420(1.0202)

147p = 50.48

p = 0.34

Probability of Price increase = 0.34

Probability of Price decrease = 0.66

$\frac{0.34 \times 75 + 0.66 \times 0}{1.0202}$ = ₹ 25.24

Yes, the value of option under both Models is same.

(ii) Basis of valuation of options :

- Binomial model uses an approach called "Risk less Hedge Approach" to find the price of the option, by creating a portfolio which will have same value at expiration irrespective of any price.

Hedge means to create an equal and opposite position for protecting the value of portfolio.

- In Risk Neutral Model, valuation of options is based on arbitrage and is therefore independent of risk preferences; one should be able to value options assuming any set of risk preferences and get the same answer.

BLACK SCHOLES MODEL

Question – 14

From the following data for certain stock, find the value of a call option:

Price of stock now	=	₹ 80
Exercise price	=	₹ 75
Standard deviation of continuously compounded annual return	=	0.40
Maturity period	=	6 months
Annual interest rate	=	12%

Given

Number of S.D. from Mean, (z)	Area of the left or right (one tail)
0.25	0.4013
0.30	0.3821
0.55	0.2912
0.60	0.2743

$e^{0.12\times0.5}$ = 1.062

Ln 1.0667 = 0.0646

(SM TYK – 25)

Solution:

Working Note 1: d_1 & d_2

$$d_1 = \frac{\text{Ln}\frac{S_0}{E} + \left(r + \frac{(\sigma)^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_1 = \frac{\text{Ln}\frac{80}{75} + \left(0.12 + \frac{(0.40)^2}{2}\right)0.5}{0.40\sqrt{0.5}}$$

$$d_1 = \frac{\text{Ln } 1.0667 + 0.10}{0.2828}$$

$$d_1 = \frac{0.0646 + 0.10}{0.2828} = 0.5820$$

$$d_2 = d1 - \sigma\sqrt{t}$$

$$= 0.5820 - 0.2828 = 0.2992$$

Working Note 2: n (d_1) & (d_2)

[BSM मे हमेशा Cumulative Area देखना है। But d_1 & d_2 negative दिया है तो फिर हम Tail Aera देखेंगे]

d_1 = 0.5820

$n(d_1)$ =

0.55	0.2912
0.60	0.2743
0.05	0.0169

$$n(d_1) = 0.2912 - \left(\frac{0.0169}{0.05} \times 0.032\right)$$

$n(d_1)$ = 0.2804

$n(d_1)$ = 1 – 0.2804 = 0.7196

d_2 = 0.2992

$n(d_2)$ =

0.25	0.4013
0.30	0.3821
0.05	0.0192

$n(d_2) = 0.4013 - \left(\frac{0.0192}{0.05} \times 0.0492\right)$

$n(d_2) = 0.3824$

$n(d_2) = 1 - 0.3824 = 0.6176$

$C_0 = S_0 \times n(d_1) - \frac{E}{e^{rt}} \times n(d_2)$

$= 80 \times 0.7196 - \frac{75}{e^{0.12 \times 0.5}} \times 0.6176$

$= 80 \times 0.7196 - \frac{75}{1.062} \times 0.6176$

= ₹ 13.952

Question – 15

The shares of TIC Ltd. are currently priced at ₹ 415 and call option exercisable in three months' time has an exercise rate of ₹ 400. Risk free interest rate is 5% p.a. and standard deviation (volatility) of the share price is 22%. The TIC Ltd. is not going to declare any dividend over the next three months.

(i) DECIDE whether the option worth buying for ₹ 25.

(ii) CALCULATE the value of aforesaid call option if the current price of share is considered as ₹ 380.

(iii) CALCULATE the value of aforesaid call option if present price of share is taken as ₹ 408 and a dividend of ₹ 10 is expected to be paid in the two months' time.

Given

In(1.0375) = 0.03681, In(0.95) = -0.05129 and In(0.9952) = -0.00481

$e^{0.0125} = 1.0126$ and $e^{0.00833} = 1.0084$

Cumulative Area of Number of S.D. from Mean (z)

Z	0.0150	0.1250	0.3933	0.5033	-0.2976	-0.4076
Area	0.5060	0.5497	0.6530	0.6926	0.3830	0.3418

(MTP Oct – 2022)

Solution:

(1) Current Price of share of TIC Ltd. = ₹ 415

Exercise rate = ₹ 400

Risk free interest rate is = 5% p.a.

SD (Volatility) = 22%

Based on the above bit is calculated value of an option based on Black Scholes Model:

$$d_1 = \frac{In\left(\frac{415}{400}\right) + \left[0.05 + \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$$

$$= \frac{0.03681 + 0.01855}{0.11} = 0.5033$$

$$d_2 = \frac{In\left(\frac{415}{400}\right) + \left[0.05 - \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$$

$$= \frac{0.03681 + 0.00645}{0.11} = 0.3933$$

$$V_o = V_s\, N(d_1) - \frac{E}{e^{rt}} N(d_2)$$

$N(d_1)$ = N (0.5033) = 0.6926

$N(d_2)$ = N (0.3933) = 0.6530

$$\text{Value of Option} = 415(0.6926) - \frac{400}{e^{(0.05)(0.25)}}(0.6530)$$

$$= 287.43 - \frac{400}{1.0126}(0.6530)$$

= 287.43 – 257.95 = ₹ 29.48

Since market price of ₹ 25 is less than ₹ 29.48 (Black Scholes Valuation Model) indicate that option is underpriced, hence worth buying.

Extra Work

Value of Put Option: अगर Value of Call दिया हुआ है तो Value of Put, Put Call Parity से Calculate किया जा सकता है।

Put Call Parity:

$S_0 + P_0 = C_0 + P.V.\ of\ EP$

$415 + P_0 = 29.48 + \frac{400}{1.0126}$

$P_0 = 9.50$

Black Scholes Model:

$\frac{E}{e^{rt}} \times N(-d_2) - S_0\, N(-d_1)$

$\frac{400}{1.0126} \times 0.347 - 415 \times 0.3074$

$P_0 = 9.50$

(2) If the current price is taken as ₹ 380 the computations are as follows:

$$d_1 = \frac{In\left(\frac{380}{400}\right) + \left[0.05 + \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$$

$$= \frac{-0.05129 + 0.01855}{0.11} = -0.2976$$

$$d_2 = \frac{In\left(\frac{380}{400}\right) + \left[0.05 - \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$$

$$= \frac{-0.05129 + 0.00645}{0.11} = -0.4076$$

$N(d_1) = N(-0.2976) = 0.3830$

$N(d_2)$ = N (- 0.4076) = 0.3418

Value of Option = $380(0.3830) - \frac{400}{e^{(0.05)(0.25)}}(0.3418)$

= $145.54 - \frac{400}{1.0126}(0.3418)$

= 145.54 – 135.02 = ₹ 10.52

(3) Since dividend is expected to be paid in two months time was we have to adjust the share price and then use Black Scholes model to value the option:

Present Value of Dividend (using continuous discounting)

= Dividend ÷ e^{rt}

= ₹ 10/1.0084 = ₹ 9.92

Adjusted price of shares is ₹ 408.00 – ₹ 9.92 = ₹ 398.08

This can be used in Black Scholes model

$d_1 = \frac{In\left(\frac{398.08}{400}\right) + \left[0.05 + \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$

$= \frac{-0.00481 + 0.01855}{0.11} = 0.1250$

$d_2 = \frac{In\left(\frac{398.08}{400}\right) + \left[0.05 - \frac{1}{2}(0.22)^2\right]0.25}{0.22\sqrt{0.25}}$

$= \frac{-0.00481 + 0.00645}{0.11} = 0.0150$

$N(d_1)$ = N (0.1250) = 0.5497

$N(d_2)$ = N (0.0150) = 0.5060

Value of Option = $398.08(0.5497) - \frac{400}{e^{(0.05)(0.25)}}(0.5060)$

$$= 218.82 - \frac{400}{1.0126}(0.5060)$$

$$= 218.82 - 199.88 = ₹\ 18.94$$

PART – 02: FUTURE

(V) MARGIN A/C

Question – 16

On 31/08/2021 Mr. R has taken a Long position of Two lots of Nifty Futures at 17300.

One lot of Nifty future is 50 units.

Initial Margin required is 10% of Contract Value.

Maintenance Margin required is 80% of Initial Margin.

The closing price of 5 days are given below –

Date	Closing Price of Nifty Future
01/09/2021	17340
02/09/2021	17180
03/09/2021	16990
06/09/2021	16900
07/09/2021	17120

You are required to-

(i) Prepare a statement showing the daily balances in the margin account & payment on margin calls, if any.

(ii) Compute the Gain or Loss of Mr. R, if contract squared off on 07/09/2021.

(iii) What would be the Gain or Loss if Mr. R, had taken the short position?

(Exam December – 2021) (8 Marks)

Solution:

Initial Margin = [17,300 × 50 × 2] × 10%

= ₹ 1,73,000

Maintenance Margin = ₹ 1,73,000 × 80%

= ₹ 1,38,400

(i) Margin Account

Date	Closing Price	Profit/Loss	Balance	Margin Call
31/08/2021	17,300	–	1,73,000	–
01/09/2021	17,340	40 × 100 = +4,000	1,77,000	–
02/09/2021	17,180	-160 × 100 = -16,000	1,61,000	–
03/09/2021	16,990	-190 × 100 = -19,000	1,42,000	–
06/09/2021	16,900	-90 × 100 = -9,000	1,73,000	40,000
07/09/2021	17,120	+220 × 100 = 22,000	1,95,000	–

(ii) Gain or Loss

Closing Balance = ₹ 1,95,000

(-) Initial Margin = ₹ 1,73,000

(-) Variation Margin = ₹ 40,000

Loss = ₹ 18,000

OR

(17,300 – 17,120) × 100 = 18,000 (Loss)

(iii) Gain or Loss (Short Position)

Margin Account

Date	Closing Price	Profit/Loss	Balance	Margin Call
31/08/2021	17,300	–	1,73,000	–
01/09/2021	17,340	-40 × 100 = -4,000	1,69,000	–
02/09/2021	17,180	160 × 100 = 16,000	1,85,000	–
03/09/2021	16,990	190 × 100 = 19,000	2,04,000	–
06/09/2021	16,900	90 × 100 = 9,000	2,13,000	–
07/09/2021	17,120	-220 × 100 = -22,000	1,91,000	–

Gain or Loss

Closing Balance = ₹ 1,91,000

(-) Initial Margin	= ₹ 1,73,000
Gain	= ₹ 18,000

Question – 17

Sensex futures are traded at a multiple of 50. Consider the following quotations of Sensex futures in the 10 trading days during February, 2009:

Day	High	Low	Closing
4-2-09	3306.40	3290.00	3296.50
5-2-09	3298.00	3262.50	3294.40
6-2-09	3256.20	3227.00	3230.40
7-2-09	3233.00	3201.50	3212.30
10-2-09	3281.50	3256.00	3267.50
11-2-09	3283.50	3260.00	3263.80
12-2-09	3315.00	3286.30	3292.00
14-2-09	3315.00	3257.10	3309.30
17-2-09	3278.00	3249.50	3257.80
18-2-09	3118.00	3091.40	3102.60

Abhishek bought one sensex futures contract on February, 04. The average daily absolute change in the value of contract is ₹ 10,000 and standard deviation of these changes is ₹ 2,000. The maintenance margin is 75% of initial margin.

You are required to determine the daily balances in the margin account and payment on margin calls, if any.

(SM TYK – 14)

Solution:

Initial Margin = $\mu + 3\sigma$

= 10,000 + (3 × 2000)

= ₹ 16,000

Maintenance Margin = 16,000 × 75%

= ₹ 12,000

Margin A/c (Long)

Day	Closing Price	Profit Loss	Margin A/c (₹)	Balance
04/02/09	3,296.50	---	-	16,000
05/02/09	3,294.40	(3,294.40 – 3,296.50) × 50 = -105	-	15,895
06/02/09	3,230.40	(3,230.40 – 3,294.40) × 50 = - 3,200	-	12,695
07/02/09	3,212.30	(3,212.30 – 3,230.40) × 50 = - 905	4210	16,000
10/02/09	3,267.50	(3,267.50 – 3,212.30) × 50 = 2,760	-	18,760
11/02/09	3,263.80	(3,263.80 – 3,267.50) × 50 = -185	-	18,575
12/02/09	3,292.00	(3,292.00 – 3,263.8) × 50 = 1,410	-	19,985
14/02/09	3,309.30	(3,309.30 – 3,292.00) × 50 = 865	-	20,850
17/02/09	3,257.80	(3,257.80 – 3,309.30) × 50 = -2,575	-	18,275
18/02/09	3,102.60	(3,102.60 – 3,257.80) × 50 = -7,760	5485	16,000

(VI) VALUATION OF FUTURE

Question – 18

The following data relate to Anand Ltd.'s share price:

Current price per share ₹ 1,800

6 months future's price/share ₹ 1,950

Assuming it is possible to borrow money in the market for transactions in securities at 12% per annum, you are required:

(i) to calculate the theoretical minimum price of a 6-months forward purchase; and

(ii) to explain arbitrate opportunity.

(SM TYK – 02)

Solution:

(1) Theoretical Future Price

F = S (1 + r) – D

= 1,800 (1.06)

= ₹ 1,908

(2) Arbitrage

(i) Action: Since future is overpriced, hence sell future & buy spot

(ii) Process

Today

- Borrow ₹ 1,800 & buy stock
- Contract to sell such stock at future price ₹ 1,950

After 6 Months

Cash Inflows	
Sell share at future price	= ₹ 1,950
Cash outflows	
Repayment of Borrowing ₹ 1,800 (1.06)	= ₹ 1,908
Arbitrage Gain	= ₹ 42

Question – 19

Calculate the price of 3 months PQR futures, if PQR (FV ₹ 10) quotes ₹ 220 on NSE and the three months future price quotes at ₹ 230 and the one month borrowing rate is given as 15 percent per annum and the expected annual dividend is 25 percent, payable before expiry. Also examine arbitrage opportunities.

(SM TYK – 04)

Solution:

(1) Theoretical Future Price